Event-triggered control and observer design for infinite-dimensional systems

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Outline

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Abstract control

- Abstract dynamical system (state $X \in \mathcal{X}$ and control $U \in \mathcal{U}$)[1]

$$\begin{cases} \frac{d}{dt}X(t) & = F\left(X(t), U(t)\right), \quad t \in \mathbb{R}_+, \\ X(0) & = X_0, \end{cases} \tag{1}$$

- Exact controllability consists in driving system from an initital state to a given targen state in a given time, i.e.

$$\forall X_0 \in \mathcal{X}, \quad \forall X_1 \in \mathcal{X}, \quad \forall T > 0, \quad \exists \left(t \in [0,T] \mapsto U(t)\right) \in \mathcal{U}, \\ X \text{ solution to } (1), \quad X(T) = X_1, \tag{2}$$

- Optimal control consists in driving system while keeping a certain cost minimal

$$\text{Find a minimiser } U_m \text{ of } C(U) := \int_0^T R(X(t), U(t))dt + P(X(T)), \tag{3}$$

among all admissible control stategies

- Asymptotic stabilisation suppose the existence of a couple $(X_e, U_e) \in \mathcal{X} \times \mathcal{U}$ such that $F(X_e, U_e) = 0$ and provide a function $\mathbb{U} : \mathcal{X} \mapsto \mathcal{U}$ such $\mathbb{U}(X_e) = U_e$ for which

$$\frac{d}{dt}X(t) = F\left(X(t), \mathbb{U}(X(t))\right), \quad t \in \mathbb{R}_+, \quad X(0) = X_0, \tag{4}$$

is asymptotically stable

[1] a key aspect of mathematical analysis consists in selecting suitable spaces $\mathcal{X}, \mathcal{U}$

Motivation

- A control system is a dynamical system on which we can **act by using suitable controls** (closed-loop) which (asymptotically) **stabilises state** we want to reach

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  $\Rightarrow$ meant to reduce communication and computation *while preserving stability*

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  $\Rightarrow$ meant to reduce communication and computation *while preserving stability*
- Building an observer
  $\Rightarrow$ *real-time simulation* of system driven by same input
  $\Rightarrow$ correction derived from difference between actual output and predicted output

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- Event-triggered control (ETC)
  $\Rightarrow$ suited for systems where *communication is prevalent*
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- Building an observer
  $\Rightarrow$ *real-time simulation* of system driven by same input
  $\Rightarrow$ correction derived from difference between actual output and predicted output
- Our goal is to **investigate stabilisation** of abstract infinite-dimensional (PDE) control system **using Luenberger observer and event-triggered updates of control**

In a nutshell

- Linear systems governed by ODE, matrices A, B, state $x(\cdot)$, control input $u(\cdot)$

$$\dot{x}(t) = Ax(t) + Bu(t), \quad t \geq 0, \tag{5}$$

[2] Tabuada 2007; Heemels, Johansson, and Tabuada 2012; Donkers and Heemels 2012.

[3] Heemels, Donkers, and Teel 2013; Yi, Johansson, and Dimarogonas 2017; Postoyan, Tabuada, Nešić, and Anta 2015.

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- In ETC[2], control input is updated *only* at discrete instants $(t_k)_{k\in\mathbb{N}}$

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bu(t_k), \quad t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}, \\ &\|x(t_k) - x(t)\|^2 < \sigma \|x(t)\|^2, \quad t \in [t_k, t_{k+1}), \quad \sigma \in (0,1), \end{aligned} \tag{6}$$

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- Under suitable assumptions, such rule ensures[3] **exponential stability** and guarantee *strictly positive inter-event times*

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- Under suitable assumptions, such rule ensures[3] **exponential stability** and guarantee *strictly positive inter-event times*
- Transition to PDE is far from trivial[4] due to unbounded operators, semigroup dynamics, coexistence of stable and unstable spectral modes, etc

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- Abstract infinite-dimensional control system of (closed-loop) form

$$\begin{cases} \dot{z}(t) = \mathcal{A}z(t) + Bu(t), & t \geq 0, \\ u(t) = Kz(t), & t \geq 0, \\ z(0) = z_0, \end{cases} \tag{7}$$

where $z_0 \in \mathcal{H}$ is initial condition of system

[5] Curtain and Weiss 2006.

Continuous closed loop system

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- Well-known[5] that if ($\mathcal{H}$ and $\mathcal{U}$ Hilbert spaces)
  1. $\mathcal{A} : \mathcal{D}(\mathcal{A}) \subset \mathcal{H} \to \mathcal{H}$ is infinitesimal generator of a C_0-semigroup $\exp(t\mathcal{A})$
  2. $B \in \mathcal{L}(\mathcal{U}, \mathcal{H})$ and $K \in \mathcal{L}(\mathcal{H}, \mathcal{U})$ are bounded linear operators
  3. $\mathcal{A} + BK$ is infinitesimal generator of an exponentially stable C_0-semigroup on $\mathcal{H}$

  then for $z_0 \in \mathcal{D}(\mathcal{A})$ closed-loop system (7) is **exponentially stable**, i.e.

$$\exists C_0 > 0, \quad \exists \lambda > 0, \quad \|z(t)\|^2_{\mathcal{H}} \leq C_0 e^{-\lambda t}, \quad \forall t \geq 0, \tag{8}$$

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Continuous closed loop system (observation-based control)

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  where $z_0 \in \mathcal{H}$ is initial condition of system
- Assuming that ($\mathcal{H}$, $\mathcal{U}$ and $\mathcal{Y}$ Hilbert spaces)
  1. $\mathcal{A} : \mathcal{D}(\mathcal{A}) \subset \mathcal{H} \to \mathcal{H}$ is infinitesimal generator of a C_0-semigroup $\exp(t\mathcal{A})$
  2. $B \in \mathcal{L}(\mathcal{U}, \mathcal{H})$ and $K \in \mathcal{L}(\mathcal{Y}, \mathcal{U})$ are bounded linear operators
  3. $C \in \mathcal{L}(\mathcal{H}, \mathcal{Y})$ is a bounded linear operator
  4. $\mathcal{A} + BKC$ is infinitesimal generator of an exponentially stable C_0-semigroup on $\mathcal{H}$

  then for $z_0 \in \mathcal{D}(\mathcal{A})$ closed-loop system (9) is **exponentially stable**, i.e.

$$\exists C_0 > 0, \quad \exists \lambda > 0, \quad \|z(t)\|^2_{\mathcal{H}} \leq C_0 e^{-\lambda t}, \quad \forall t \geq 0, \tag{10}$$

ETC design

- Control strategy where updates happen only when a certain event occurs
- Event is usually defined by a threshold, on system state or error
- **Reduces unnecessary communication** and computation compared with periodic control (periodic control acts on a schedule while ETC acts only when needed)

ETC system

- LB, SE[6] established exponential stability of control system (7) when control is based on an observation operator and subjected to aperiodic sample-and-hold mechanism

$$\begin{cases} \dot{z}(t) = \mathcal{A}z(t) + Bu(t_k) & t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}, \\ u(t) = Ky(t), & t \geq 0, \\ y(t) = Cz(t), & t \geq 0, \\ z(0) = z_0, \end{cases} \tag{11}$$

where $z_0 \in \mathcal{H}$, subjected to triggering mechanism

$$t_{k+1} := \sup \left\{ t > t_k, \quad \|Cz(t) - Cz(t_k)\|_{\mathcal{H}} \leq \gamma \|z(t)\|_{\mathcal{H}} \right\}, \quad \gamma > 0, \tag{12}$$

[6] Baudouin and Ervedoza 2025.

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- Assuming that ($\mathcal{H}$, $\mathcal{U}$ and $\mathcal{Y}$ Hilbert spaces)
  1. $\mathcal{A} : \mathcal{D}(\mathcal{A}) \subset \mathcal{H} \to \mathcal{H}$ is infinitesimal generator of a C_0-semigroup $\exp(tA)$
  2. $B \in \mathcal{L}(\mathcal{U}, \mathcal{H})$ and $K \in \mathcal{L}(\mathcal{Y}, \mathcal{U})$ are bounded linear operators
  3. $C \in \mathcal{L}(\mathcal{H}, \mathcal{Y})$ is a bounded linear operator
  4. $\mathcal{A} + BKC$ is infinitesimal generator of an exponentially stable C_0-semigroup on $\mathcal{H}$
  5. $\mathcal{A}$ is skew-adjoint (i.e. $\mathcal{A}^* = -\mathcal{A}$)

then for $z_0 \in \mathcal{D}(\mathcal{A})$ ETC system (11)-(12) is **exponentially stable** i.e.

$$\exists C_0 > 0, \quad \exists \lambda > 0, \quad \|z(t)\|_{\mathcal{H}}^2 \leq C_0 e^{-\lambda t}, \quad \forall t \geq 0, \tag{13}$$

[6] Baudouin and Ervedoza 2025.

- State-feedback control requires value (at each time) that **approximates**[7] system state
- **Hardware sensors** are physical instruments that directly measure state variables
  $\Rightarrow$ sometimes expensive, difficult to implement, or unable to measure all state
- **Software sensors** algorithms *based on a model and available physical measurements*
  $\Rightarrow$ provide online estimates of unmeasured state and implementable in practice

[7] Girard 2015b; Etienne, Di Gennaro, and Barbot 2015; Zhang and Feng 2014.

- Observer design control system

$$
\begin{cases}
\dot{z}(t) = \mathcal{A}z(t) + Bu(t), & t \geq 0, \\
u(t) = K\hat{z}(t), & t \geq 0, \\
\dot{\hat{z}}(t) = \mathcal{A}\hat{z}(t) + Bu(t) + L(C\hat{z}(t) - y(t)), & t \geq 0, \\
y(t) = Cz(t), & t \geq 0, \\
z(0) = z_0, \quad \hat{z}(0) = \hat{z}_0,
\end{cases}
\tag{14}
$$

where $(z_0, \hat{z}_0) \in \mathcal{H} \times \mathcal{H}$ is initial condition of system

Observer design continuous system

- Observer design control system

$$\begin{cases} \dot{z}(t) = \mathcal{A}z(t) + Bu(t), & t \geq 0, \\ u(t) = K\hat{z}(t), & t \geq 0, \\ \dot{\hat{z}}(t) = \mathcal{A}\hat{z}(t) + Bu(t) + L(C\hat{z}(t) - y(t)), & t \geq 0, \\ y(t) = Cz(t), & t \geq 0, \\ z(0) = z_0, \quad \hat{z}(0) = \hat{z}_0, \end{cases} \tag{14}$$

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  4. $C \in \mathcal{L}(\mathcal{H}, \mathcal{Y})$ and $L \in \mathcal{L}(\mathcal{Y}, \mathcal{H})$ are bounded linear operators
  5. $\mathcal{A} + LC$ is infinitesimal generator of an exponentially stable C_0-semigroup on $\mathcal{H}$
  6. $\mathcal{A}$ is skew-adjoint

then for $(z_0, \hat{z}_0) \in \mathcal{D}(\mathcal{A}) \times \mathcal{D}(\mathcal{A})$ control system (14) is **exponentially stable**, i.e.

$$\exists C_0 > 0, \quad \exists \lambda > 0, \quad \|z(t)\|^2_{\mathcal{H}} + \|\hat{z}(t)\|^2_{\mathcal{H}} \leq C_0 e^{-\lambda t}, \quad \forall t \geq 0, \tag{15}$$

- ETC observer design control system

$$\begin{cases} \dot{z}(t) = \mathcal{A}z(t) + Bu(t_k), & t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}, \\ u(t) = K\hat{z}(t), & t \geq 0, \\ \dot{\hat{z}}(t) = \mathcal{A}\hat{z}(t) + Bu(t_k) + L(C\hat{z}(t) - y(t)), & t \geq 0, \\ y(t) = Cz(t), & t \geq 0, \\ z(0) = z_0, \quad \hat{z}(0) = \hat{z}_0, \end{cases} \tag{16}$$

where $(z_0, \hat{z}_0) \in \mathcal{H} \times \mathcal{H}$ is initial condition of system subjected to

$$t_{k+1} := \sup\left\{t > t_k, \quad \|\hat{z}(t) - \hat{z}(t_k)\|_{\mathcal{H}} \leq \gamma \|\hat{z}(t)\|_{\mathcal{H}}\right\}, \quad \gamma > 0, \tag{17}$$

ETC and observer design system

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- Assuming that ($\mathcal{H}$, $\mathcal{U}$ and $\mathcal{Y}$ Hilbert spaces)
  1. $\mathcal{A} : \mathcal{D}(\mathcal{A}) \subset \mathcal{H} \to \mathcal{H}$ is infinitesimal generator of a C_0-semigroup $\exp(t\mathcal{A})$
  2. $B \in \mathcal{L}(\mathcal{U}, \mathcal{H})$ and $K \in \mathcal{L}(\mathcal{H}, \mathcal{U})$ are bounded linear operators
  3. $\mathcal{A} + BK$ is infinitesimal generator of an exponentially stable C_0-semigroup on $\mathcal{H}$
  4. $C \in \mathcal{L}(\mathcal{H}, \mathcal{Y})$ and $L \in \mathcal{L}(\mathcal{Y}, \mathcal{H})$ are bounded linear operators
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  6. $\mathcal{A}$ is skew-adjoint

then for $(z_0, \hat{z}_0) \in \mathcal{D}(\mathcal{A}) \times \mathcal{D}(\mathcal{A})$ **IS** control system (16) **exponentially stable ?**, i.e.

$$\textbf{?}\, (\exists C_0 > 0, \quad \exists \lambda > 0), \quad \|z(t)\|^2_{\mathcal{H}} + \|\hat{z}(t)\|^2_{\mathcal{H}} \leq C_0 e^{-\lambda t}, \quad \forall t \geq 0, \tag{18}$$

Dynamic triggering rule

- To design triggering mechanism that determines $(t_k)_{k\in\mathbb{N}}$, can not consider

$$t_{k+1} := \sup\left\{t > t_k, \quad \|\hat{z}(\tau) - \hat{z}(t_k)\|_{\mathcal{H}} \leq \gamma\|\mathbf{z}(\tau)\|_{\mathcal{H}} + \gamma\|\hat{z}(\tau)\|_{\mathcal{H}}, \quad \forall\tau \in [t_k, t)\right\}, \tag{19}$$

since it requires knowledge of **true state** $z(\cdot)$ **assumed unmeasured**

[8] Girard 2015a; Yi, Johansson, and Dimarogonas 2017; Koudohode, Baudouin, and Tarbouriech 2022.

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- *Could use static triggering condition* that directly leverages observer state $\hat{z}(\cdot)$

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implementable in practice, as it depends only on observer state $\hat{z}(\cdot)$

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- *To increase flexibility of triggering mechanism* and to **achieve better control of inter-event times**, we equip system (16) with internal scalar dynamic[8] variable $m(\cdot)$

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- Introduce maximal time T^* under which system (16)-(21) has a solution

$$T^* := \begin{cases} \infty, & \text{if card}\left((t_k)_{k\in\mathbb{N}}\right) < \infty, \\ \limsup_k t_k, & \text{otherwise} \end{cases} \tag{22}$$

[8] Girard 2015a; Yi, Johansson, and Dimarogonas 2017; Koudohode, Baudouin, and Tarbouriech 2022.

Key assumptions

$\mathcal{H}$, $\mathcal{U}$ and $\mathcal{Y}$ are Hilbert spaces

Assumptions (G)

- *$\mathcal{A} : \mathcal{D}(\mathcal{A}) \subset \mathcal{H} \to \mathcal{H}$ is infinitesimal generator of a C_0-semigroup* $\exp(t\mathcal{A})$
- *$B \in \mathcal{L}(\mathcal{U}, \mathcal{H})$ and $K \in \mathcal{L}(\mathcal{H}, \mathcal{U})$ are bounded linear operators*
- *$\mathcal{A} + BK$ is infinitesimal generator of an exponentially stable C_0-semigroup on $\mathcal{H}$*
- *$C \in \mathcal{L}(\mathcal{H}, \mathcal{Y})$ and $L \in \mathcal{L}(\mathcal{Y}, \mathcal{H})$ are bounded linear operators*
- *$\mathcal{A} + LC$ is infinitesimal generator of an exponentially stable C_0-semigroup on $\mathcal{H}$*
- *$\mathcal{A}$ satisfies quasi-dissipativity condition*

$$\exists c_{\mathcal{A}} \geq 0, \quad \forall z_0 \in \mathcal{D}(\mathcal{A}), \quad \mathrm{Re} \, \langle \mathcal{A} z_0, z_0 \rangle_{\mathcal{H}} \leq c_{\mathcal{A}} \left\| z_0 \right\|_{\mathcal{H}}^2 . \tag{23}$$

Assumption (M)

- $m \in C^0(\mathbb{R}_+; \mathbb{R}_+)$,
- $\forall T > 0, \quad \exists \delta_T > 0, \quad \inf_{t \in [0,T]} m(t) \geq \delta_T$,
- $\exists C_m > 0, \quad \exists \mu > 0, \quad m(t) \leq C_m e^{-\mu t}, \quad \forall t \geq 0$,

Augmented state $Z = (\hat{z}, e)^\top \in \mathcal{H} \times \mathcal{H}$ satisfies

$$\begin{cases} \dot{Z}(t) = \mathcal{A}_{\mathrm{aug}} Z(t) + \hat{d}_k(t), & t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}, \\ Z(0) = Z_0, \end{cases} \tag{24}$$

$$\mathcal{A}_{\mathrm{aug}} := \begin{pmatrix} \mathcal{A} + BK & LC \\ 0 & \mathcal{A} + LC \end{pmatrix}, \quad \mathcal{D}(\mathcal{A}_{\mathrm{aug}}) = \mathcal{D}(\mathcal{A}) \times \mathcal{D}(\mathcal{A}), \tag{25}$$

$$\hat{d}_k(t) := \begin{pmatrix} BK(\hat{z}(t_k) - \hat{z}(t)) \\ 0 \end{pmatrix}, \quad t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}, \tag{26}$$

Lemma (Augmented closed-loop generator)

Under Assumptions (G), $\mathcal{A}_{\mathrm{aug}}$ generates a C_0-semigroup on $\mathcal{H} \times \mathcal{H}$.

Theorem (Well-posedness of a maximal solution)

Under Assumptions (G) and Assumption (M), there exist a strictly increasing sequence $(t_k)_{k\in\mathbb{N}} \subset \mathbb{R}_+$ with $t_0 = 0$ and a **unique mild solution**

$$Z \in C([0, T^*); \mathcal{H} \times \mathcal{H}), \tag{27}$$

where $T^* = \sup_k t_k \in (0, \infty]$, such that

$$Z(t) = \exp((t - t_k)\mathcal{A}_{\mathrm{aug}})Z(t_k) + \int_{t_k}^{t} \exp((t - s)\mathcal{A}_{\mathrm{aug}}) \begin{pmatrix} BK\hat{z}(t_k) \\ 0 \end{pmatrix} ds, \quad \forall t \in [t_k, t_{k+1}), \tag{28}$$

and triggering rule (21) holds on each interval $[t_k, t_{k+1})$ for $k \in \mathbb{N}$,

Key points

- *Existence and uniqueness* despite **implicit coupling** (t_{k+1} depends on the solution which itself depends on t_{k+1})
- Sequence $(t_k)_{k\in\mathbb{N}}$ is constructed *simultaneously* with solution by induction (no a priori knowledge of trigger times is needed)

Regularity of solutions

Theorem (Piecewise classical regularity)

Under Assumptions (G) *and Assumption* (M), *and if in addition* $Z_0 \in \mathcal{D}(\mathcal{A}) \times \mathcal{D}(\mathcal{A})$ *then* $Z(\cdot)$ *is* **weakly continuous** *in* $[0, T^*)$ *with values in* $\mathcal{D}(\mathcal{A}) \times \mathcal{D}(A)$ *and* **piecewise classical** *in* $[0, T^*)$ *with values in* $\mathcal{H} \times \mathcal{H}$, *i.e.*

$$Z \in C_w((t_k, t_{k+1}); \mathcal{D}(\mathcal{A}) \times \mathcal{D}(\mathcal{A})) \cap C^1((t_k, t_{k+1}); \mathcal{H} \times \mathcal{H}), \quad \forall k \in \mathbb{N}. \tag{29}$$

Moreover, it holds [a] *that for all* $t \in [0, T^*)$,

$$\left\|\dot{\hat{z}}\right\|_{L^\infty(0,t;\mathcal{H})} \leq \frac{M_{\mathcal{A}}}{m_{\mathcal{A}}} e^{(\|BK\|_{\mathcal{A}} + c_{\mathcal{A}})t} \left(\|(\mathcal{A} + BK)\hat{z}_0 + LCe_0\|_{\mathcal{H}} + t \|LC\|_{\mathcal{L}(\mathcal{H})} M_L(t) \|(\mathcal{A} + LC)e_0\|_{\mathcal{H}} \right) \tag{30}$$

[a] where $c_{\mathcal{A}}, m_{\mathcal{A}}, M_{\mathcal{A}}$ are explicit positive constants and an equivalent norm $\|\cdot\|_{\mathcal{A}}$ and

$$\|BK\|_{\mathcal{A}} := \sup_{z \neq 0} \frac{\|BKz\|_{\mathcal{A}}}{\|z\|_{\mathcal{A}}}, \quad M_L(t) := \sup_{0 \leq s \leq t} \left\| e^{s(\mathcal{A}+LC)} \right\|_{\mathcal{L}(\mathcal{H})},$$

Key points

- Regularity is *piecewise*: $u = K\hat{z}(t_k)$ is frozen creating a jump in $\dot{\hat{z}}(\cdot)$ but not in $\hat{z}(\cdot)$
- Bound on $\|\dot{\hat{z}}\|_{L^\infty}$ grows in t

Exclusion of Zeno

Theorem (No zeno)

Under Assumptions (G) *and Assumption* (M), *solution to system* (**??**) *subjected to triggering rule* (21) *satisfies*

$$T^* = \infty, \tag{31}$$

and thus does **not experience any Zeno phenomenon**

Key points

- Zeno behaviour (infinitely many triggers in finite time) would make system *physically unrealisable*
- Dynamic variable $m(\cdot) > 0$ is *essential ingredient*: it **prevents trigger threshold from collapsing to zero** even as $\hat{z}(t) \to 0$
- Without $m(\cdot)$, Zeno *can* occur for classical static triggers as $\hat{z}(t) \to 0$

Exponential stability

Theorem (Event-triggered stability)

Choosing small enough γ, under Assumptions (G)-(M), then event-triggered system (16)-(21) is **exponentially stable** *in $\mathcal{H} \times \mathcal{H}$ for an initial datum in $\mathcal{D}(\mathcal{A}) \times \mathcal{D}(\mathcal{A})$, i.e.*

$$\exists \hat{C} > 0, \quad \exists \hat{\lambda} > 0, \quad \|\hat{z}(t)\|_{\mathcal{H}}^2 + \|e(t)\|_{\mathcal{H}}^2 \leq \hat{C}\left(\|\hat{z}_0\|_{\mathcal{H}} + \|e_0\|_{\mathcal{H}} + m_0\right) e^{-\hat{\lambda} t}, \quad \forall t \geq 0, \tag{32}$$

Key points

- Rate $\hat{\lambda} = \min(2\beta_1, 2\mu, 2\alpha_0)$ balances three competing decays: closed-loop, dynamic variable $m(\cdot)$ and observer error
- Condition on γ is **explicit**:
  larger $\gamma \Rightarrow$ fewer triggers $\Rightarrow$ smaller $\beta_1 \Rightarrow$ slower convergence
  $\Rightarrow$ *trade-off* between communication rate and stability rate
- Term m_0 in bound (32) captures initial trigger threshold:
  $\Rightarrow$ setting $m_0 \to 0$ recovers continuous-time rate

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Dynamic Internal Variable $m(\cdot)$ — ODE Instance

Trigger condition

$$\|\hat{z}(\tau)-\hat{z}(t_k)\|_{\mathcal{H}} \;\le\; \gamma\,\|\hat{z}(\tau)\|_{\mathcal{H}} + m(\tau), \quad t\in[t_k,t_{k+1}),$$

Instance — ODE $m(\cdot)$

$$\begin{cases} \dot{m}(t) = -\mu\, m(t) + \zeta\,\|C\hat{z}(t)-y(t)\|_{\mathcal{Y}}\,, \quad t\ge 0,\\ m(0)=m_0>0, \end{cases}$$

Design constraints

$$\mu\in(0,\,\alpha_0), \qquad \zeta\ge 0, \qquad m_0>0,$$

Theoretical decay rate

$$\lambda^* = \min\bigl(2\alpha_1,\; 2\mu,\; 2\alpha_0\bigr),$$

α_1: rate of $\mathcal{A}+\widetilde{\mathcal{B}}K$ $\quad\alpha_0$: rate of $\mathcal{A}+L\mathcal{C}$

Finite-dimensional case

ODE model

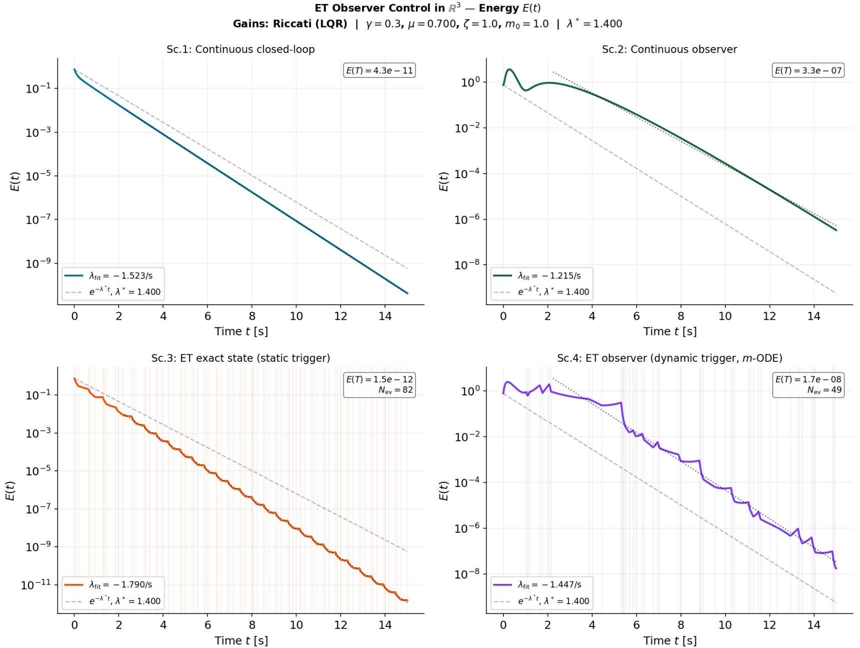
$$\begin{cases} \dot{x} = Ax + Bu(t_k), \\ y = Cx, \end{cases} \tag{33}$$

$$A = \begin{pmatrix} 0.3 & 0.6 & 0.4 \\ 0.6 & -1.0 & 0.6 \\ 0.4 & 0.6 & 0.3 \end{pmatrix}, \quad B = e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad C = e_3^\top = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \tag{34}$$

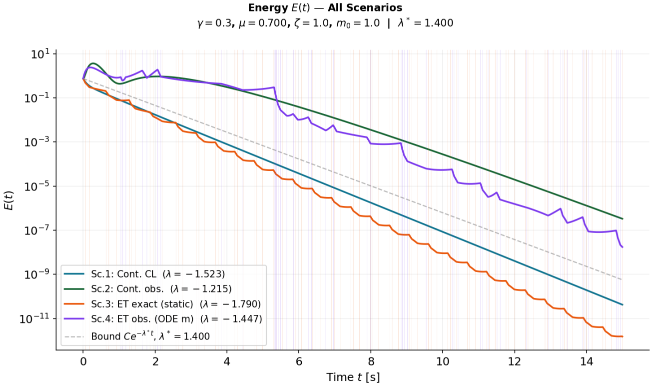
Two unstable modes $\Rightarrow$ control is necessary

Parameters

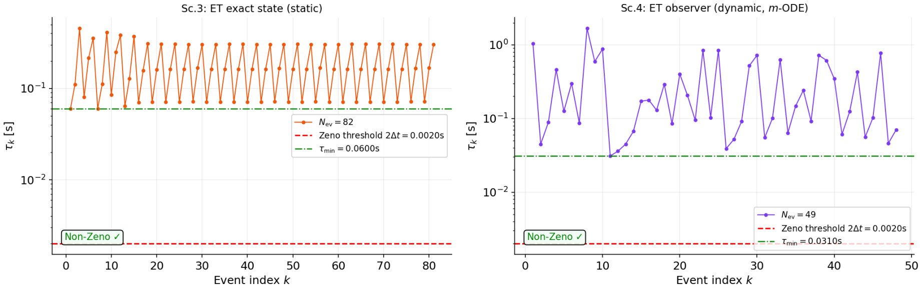
Description	Symbol	Value
Design rule	γ	0.30
$m(\cdot)$ rate	μ	0.70
Coupling	ζ	1.0
Initial state	z_0	$(1, 0.5, -0.5)$
Initial observer	$\hat{z}_0$	$(0, 0, 0)$
Initial internal	m_0	1.0
Time horizon	T	$15\,\mathrm{s}$
Time step	Δt	$10^{-3}\,\mathrm{s}$

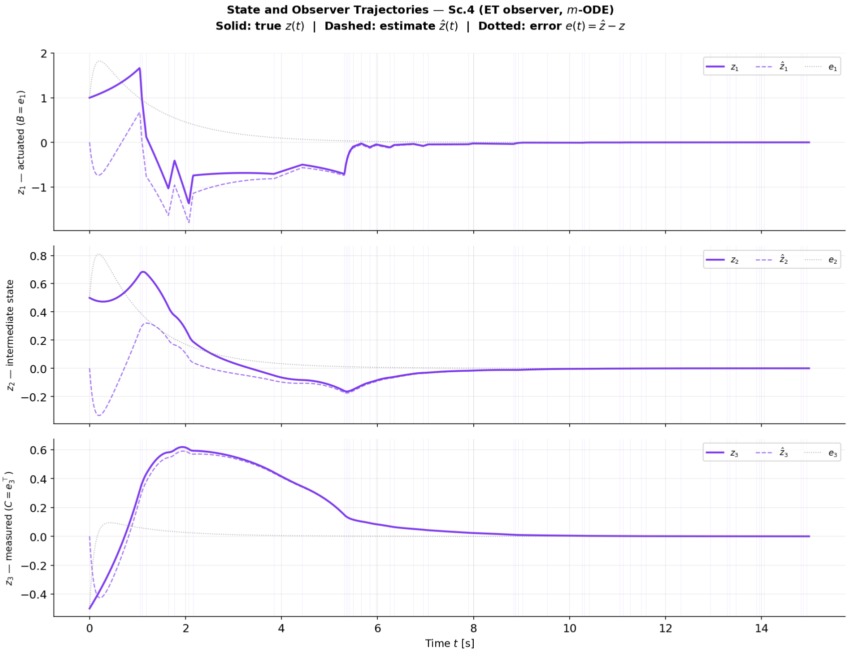


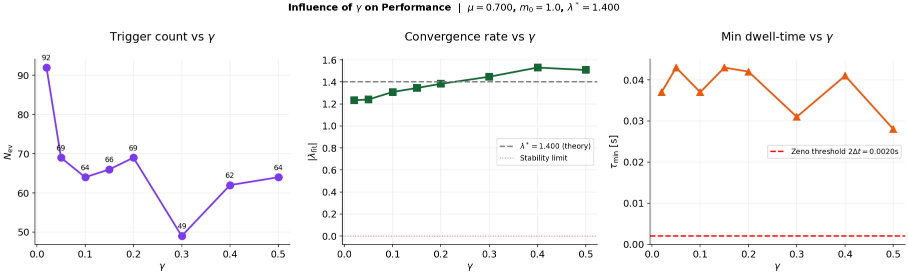
ET Observer Control in $\mathbb{R}^3$ — Energy $E(t)$
Gains: Riccati (LQR) | $\gamma = 0.3$, $\mu = 0.700$, $\zeta = 1.0$, $m_0 = 1.0$ | $\lambda^* = 1.400$
Sc.1: Continuous closed-loop
$E(T) = 4.3e-11$
$\lambda_{\text{fit}} = -1.523/\text{s}$
$e^{-\lambda^* t}$, $\lambda^* = 1.400$
Sc.2: Continuous observer
$E(T) = 3.3e-07$
$\lambda_{\text{fit}} = -1.215/\text{s}$
$e^{-\lambda^* t}$, $\lambda^* = 1.400$
Sc.3: ET exact state (static trigger)
$E(T) = 1.5e-12$
$N_{\text{ev}} = 82$
$\lambda_{\text{fit}} = -1.790/\text{s}$
$e^{-\lambda^* t}$, $\lambda^* = 1.400$
Sc.4: ET observer (dynamic trigger, m-ODE)
$E(T) = 1.7e-08$
$N_{\text{ev}} = 49$
$\lambda_{\text{fit}} = -1.447/\text{s}$
$e^{-\lambda^* t}$, $\lambda^* = 1.400$
$E(t)$
Time t [s]


Energy $E(t)$ — **All Scenarios**
$\gamma = 0.3$, $\mu = 0.700$, $\zeta = 1.0$, $m_0 = 1.0$ | $\lambda^* = 1.400$
10^{1}
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
$E(t)$
0
2
4
6
8
10
12
14
Time t [s]
Sc.1: Cont. CL ($\lambda = -1.523$)
Sc.2: Cont. obs. ($\lambda = -1.215$)
Sc.3: ET exact (static) ($\lambda = -1.790$)
Sc.4: ET obs. (ODE m) ($\lambda = -1.447$)
Bound $Ce^{-\lambda^* t}$, $\lambda^* = 1.400$

Inter-Event Intervals $\tau_k = t_{k+1} - t_k$ — No-Zeno Verification




State and Observer Trajectories — Sc.4 (ET observer, m-ODE)
Solid: true z(t) | Dashed: estimate ẑ(t) | Dotted: error e(t) = ẑ − z
z1 — actuated (B = e1)
z2 — intermediate state
z3 — measured (C = e3ᵀ)
Time t [s]


Influence of γ on Performance | μ = 0.700, m0 = 1.0, λ* = 1.400
Trigger count vs γ
Convergence rate vs γ
Min dwell-time vs γ
λ* = 1.400 (theory)
Stability limit
Zeno threshold 2Δt = 0.0020s

PDE model on $(0,1)$

$$\begin{cases} \partial_{tt} z = c^2 \partial_{xx} z - d\,\partial_t z + b(x)\,u(t_k), \\ \quad z\big|_{\{0,1\}} = 0, \end{cases} \tag{35}$$

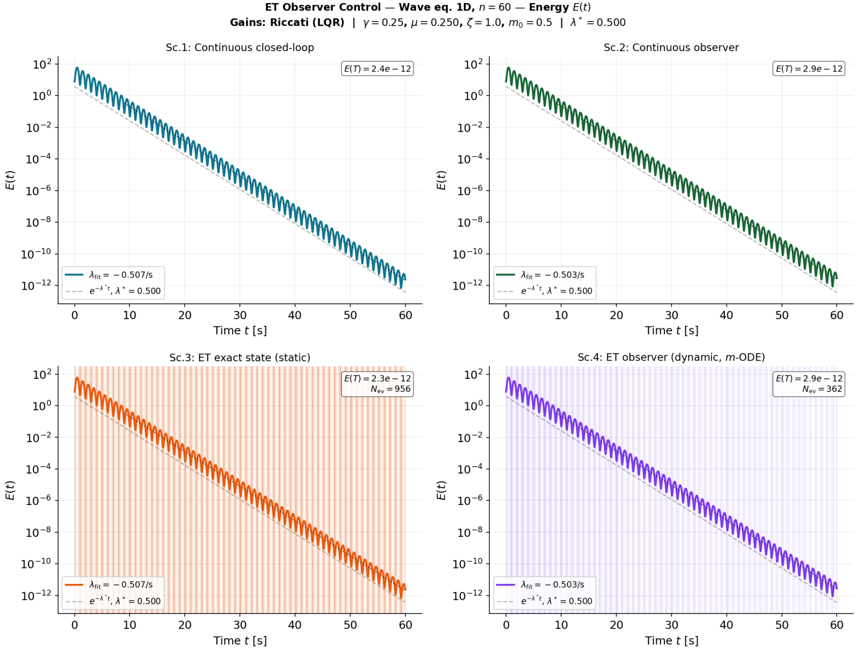
First-order formulation
State $\mathbf{z} = (u, v)^\top \in L^2(0,1) \times L^2(0,1)$,
$v = \partial_t z$,

$$\dot{\mathbf{z}} = \underbrace{\begin{pmatrix} 0 & I \\ c^2\partial_{xx} & -dI \end{pmatrix}}_{\mathcal{A}} \mathbf{z} + \underbrace{\begin{pmatrix} 0 \\ b \end{pmatrix}}_{\widetilde{\mathcal{B}}} u(t_k),$$

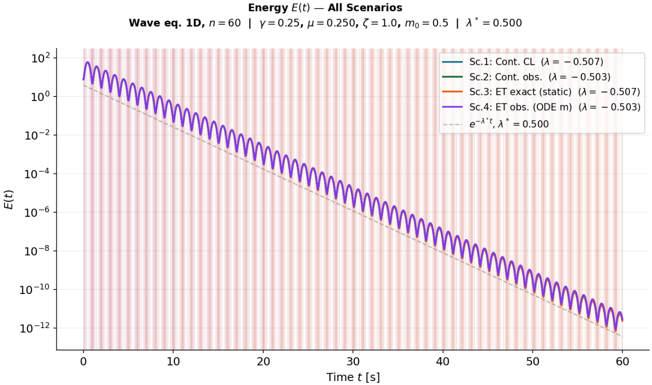
Operator	Expression	Role
$\mathcal{A}$	$\begin{pmatrix} 0 & I \\ c^2\partial_{xx} & -dI \end{pmatrix}$	Generator
$\widetilde{\mathcal{B}}$	$\begin{pmatrix} 0 \\ \delta_{x_a} \end{pmatrix}$	Point force on v
$\mathcal{C}$	$\delta_{x_o} \otimes (1,0)$	Point meas. of u

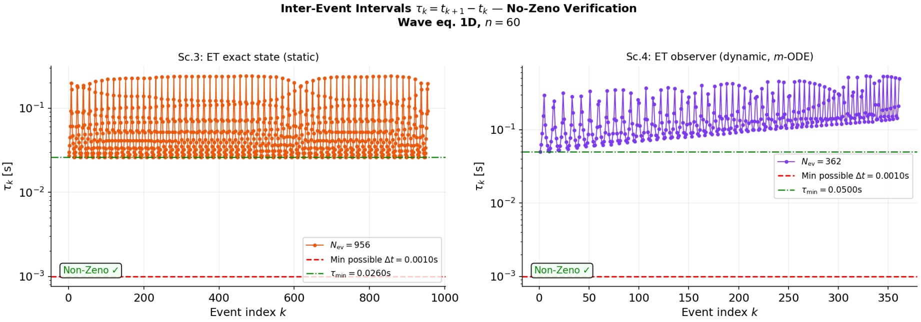
Parameters

Description	Symbol	Value
Wave speed	c	1.0
Damping	d	0.5
Actuation	x_a	≈ 0
Observation	x_o	≈ 1
Design rule	γ	0.25
$m(\cdot)$ rate	μ	0.25
Initial internal	m_0	0.5
Time horizon	T	$60\,\mathrm{s}$
Time step	Δt	$10^{-3}\,\mathrm{s}$

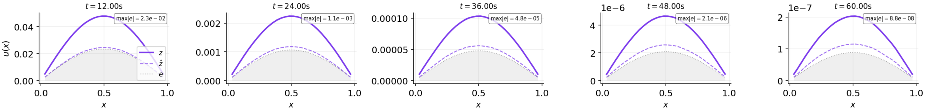


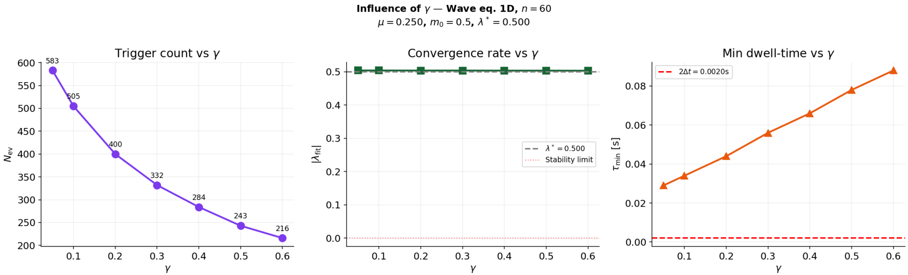
ET Observer Control — Wave eq. 1D, $n = 60$ — Energy $E(t)$
Gains: Riccati (LQR) | $\gamma = 0.25$, $\mu = 0.250$, $\zeta = 1.0$, $m_0 = 0.5$ | $\lambda^* = 0.500$
Sc.1: Continuous closed-loop
$E(T) = 2.4e-12$
$\lambda_{\text{fit}} = -0.507$/s
$e^{-\lambda^* t}$, $\lambda^* = 0.500$
Sc.2: Continuous observer
$E(T) = 2.9e-12$
$\lambda_{\text{fit}} = -0.503$/s
$e^{-\lambda^* t}$, $\lambda^* = 0.500$
Sc.3: ET exact state (static)
$E(T) = 2.3e-12$
$N_{\text{ev}} = 956$
$\lambda_{\text{fit}} = -0.507$/s
$e^{-\lambda^* t}$, $\lambda^* = 0.500$
Sc.4: ET observer (dynamic, m-ODE)
$E(T) = 2.9e-12$
$N_{\text{ev}} = 362$
$\lambda_{\text{fit}} = -0.503$/s
$e^{-\lambda^* t}$, $\lambda^* = 0.500$
$E(t)$
Time t [s]


Energy $E(t)$ — All Scenarios
Wave eq. 1D, $n = 60$ | $\gamma = 0.25$, $\mu = 0.250$, $\zeta = 1.0$, $m_0 = 0.5$ | $\lambda^* = 0.500$
$E(t)$
Time t [s]
Sc.1: Cont. CL ($\lambda = -0.507$)
Sc.2: Cont. obs. ($\lambda = -0.503$)
Sc.3: ET exact (static) ($\lambda = -0.507$)
Sc.4: ET obs. (ODE m) ($\lambda = -0.503$)
$e^{-\lambda^* t}$, $\lambda^* = 0.500$


Inter-Event Intervals $\tau_k = t_{k+1} - t_k$ — No-Zeno Verification
Wave eq. 1D, $n = 60$
Sc.3: ET exact state (static)
τ_k [s]
Event index k
$N_{ev} = 956$
Min possible $\Delta t = 0.0010$s
$\tau_{min} = 0.0260$s
Non-Zeno ✓
Sc.4: ET observer (dynamic, m-ODE)
τ_k [s]
Event index k
$N_{ev} = 362$
Min possible $\Delta t = 0.0010$s
$\tau_{min} = 0.0500$s
Non-Zeno ✓

Spatial Snapshots — Wave eq. 1D — Sc.4 (ET observer, m-ODE)
Displacement u only (velocity v omitted: boundary artefact from point observer)
Solid: true z | Dashed: estimate $\hat{z}$ | Dotted: error $e = \hat{z} - z$





Influence of γ — Wave eq. 1D, n = 60
μ = 0.250, m0 = 0.5, λ* = 0.500
Trigger count vs γ
583
505
400
332
284
243
216
N_ev
γ
Convergence rate vs γ
λ* = 0.500
Stability limit
|λ_fit|
γ
Min dwell-time vs γ
2Δt = 0.0020s
τ_min [s]
γ

PDE model on $\Omega = (0,1)^2$

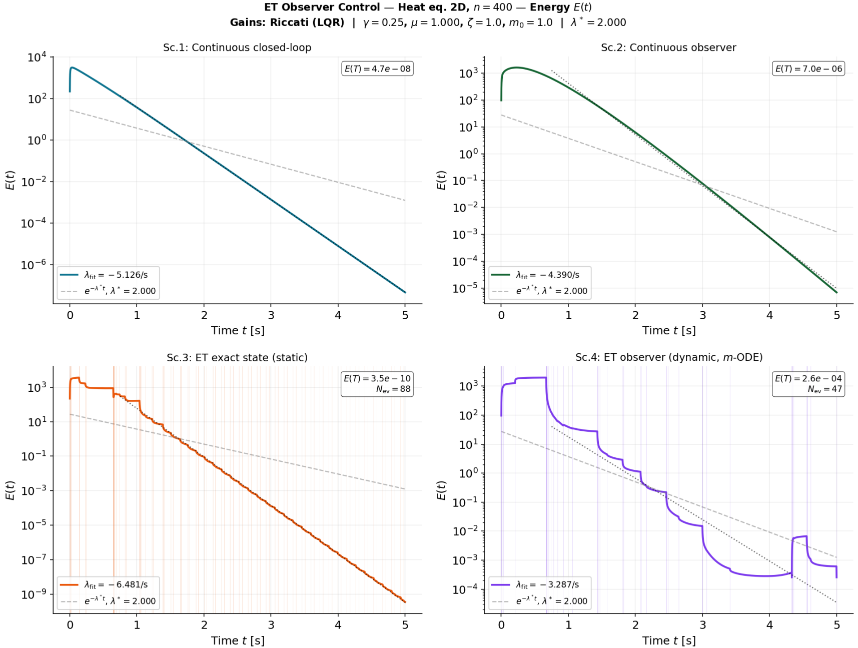
$$\begin{cases} \partial_t z = \underbrace{\kappa\Delta + \sigma I}_{\mathcal{A}}\, z + \underbrace{b(\mathbf{x})}_{\widetilde{\mathcal{B}}}\, u(t_k), \\ z\big|_{\partial\Omega} = 0, \end{cases} \tag{36}$$

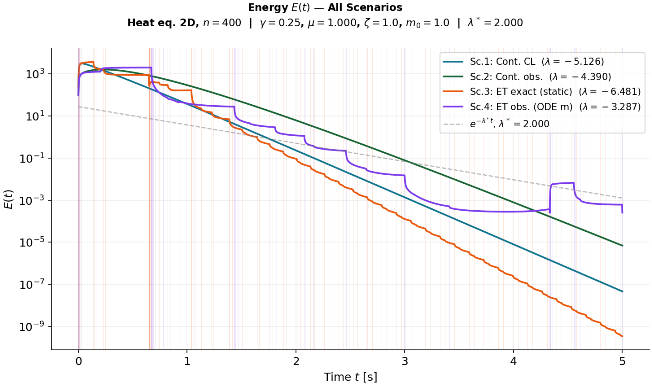
Operator	Expression	Space
$\mathcal{A}$	$\kappa\Delta + \sigma I$	$L^2(\Omega) \to L^2(\Omega)$
$\widetilde{\mathcal{B}}$	$\delta_{\mathbf{x}_a}$	$\mathbb{R} \to L^2(\Omega)$
$\mathcal{C}$	$\delta_{\mathbf{x}_o}^\top$	$L^2(\Omega) \to \mathbb{R}$

$\max \mathrm{Re}(\mathcal{A}) \approx +1.58$ *(unstable)*

Parameters

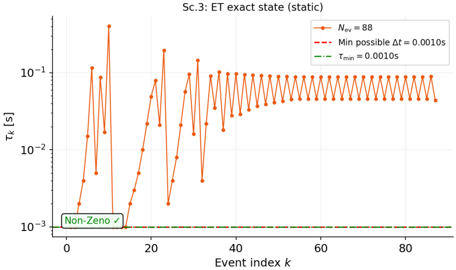
Description	Symbol	Value
Diffusivity	κ	0.25
Reaction	σ	7.50
Mesh	$n_1 \times n_2$	20×20
Actuation	$\mathbf{x}_a$	corner $(0,0)$
Observation	$\mathbf{x}_o$	corner $(1,1)$
Design rule	γ	0.20
$m(\cdot)$ rate	μ	1.0
Initial internal	m_0	1.0
Time horizon	T	$5\,\mathrm{s}$
Time step	Δt	$10^{-3}\,\mathrm{s}$

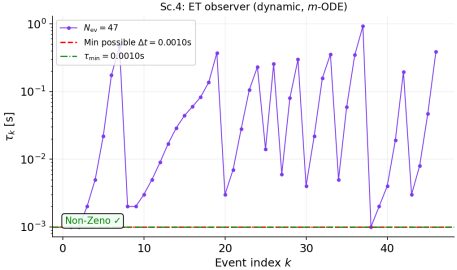

ET Observer Control — Heat eq. 2D, $n = 400$ — Energy $E(t)$
Gains: Riccati (LQR) | $\gamma = 0.25$, $\mu = 1.000$, $\zeta = 1.0$, $m_0 = 1.0$ | $\lambda^* = 2.000$
Sc.1: Continuous closed-loop
$E(T) = 4.7e-08$
$\lambda_{\text{fit}} = -5.126/\text{s}$
$e^{-\lambda^* t}$, $\lambda^* = 2.000$
Sc.2: Continuous observer
$E(T) = 7.0e-06$
$\lambda_{\text{fit}} = -4.390/\text{s}$
$e^{-\lambda^* t}$, $\lambda^* = 2.000$
Sc.3: ET exact state (static)
$E(T) = 3.5e-10$
$N_{\text{ev}} = 88$
$\lambda_{\text{fit}} = -6.481/\text{s}$
$e^{-\lambda^* t}$, $\lambda^* = 2.000$
Sc.4: ET observer (dynamic, m-ODE)
$E(T) = 2.6e-04$
$N_{\text{ev}} = 47$
$\lambda_{\text{fit}} = -3.287/\text{s}$
$e^{-\lambda^* t}$, $\lambda^* = 2.000$
$E(t)$
Time t [s]

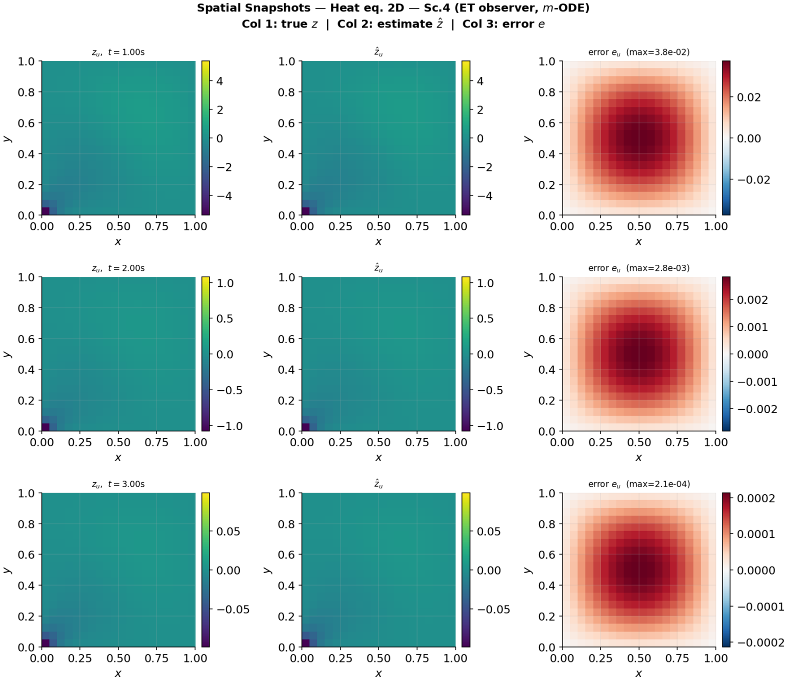


Energy $E(t)$ — All Scenarios
Heat eq. 2D, $n = 400$ | $\gamma = 0.25$, $\mu = 1.000$, $\zeta = 1.0$, $m_0 = 1.0$ | $\lambda^* = 2.000$
Sc.1: Cont. CL ($\lambda = -5.126$)
Sc.2: Cont. obs. ($\lambda = -4.390$)
Sc.3: ET exact (static) ($\lambda = -6.481$)
Sc.4: ET obs. (ODE m) ($\lambda = -3.287$)
$e^{-\lambda^* t}$, $\lambda^* = 2.000$
$E(t)$
10^{3}
10^{1}
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
0
1
2
3
4
5
Time t [s]

Inter-Event Intervals $\tau_k = t_{k+1} - t_k$ — No-Zeno Verification
Heat eq. 2D, $n = 400$






Spatial Snapshots — Heat eq. 2D — Sc.4 (ET observer, m-ODE)
Col 1: true z | Col 2: estimate ẑ | Col 3: error e
z_u, t = 1.00s
ẑ_u
error e_u (max=3.8e-02)
z_u, t = 2.00s
ẑ_u
error e_u (max=2.8e-03)
z_u, t = 3.00s
ẑ_u
error e_u (max=2.1e-04)
x
y

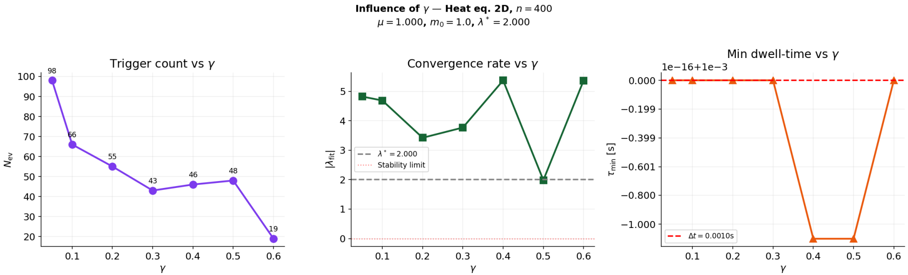

Influence of γ — Heat eq. 2D, n = 400
μ = 1.000, m0 = 1.0, λ* = 2.000
Trigger count vs γ
Nev
γ
98
66
55
43
46
48
19
Convergence rate vs γ
|λfit|
λ* = 2.000
Stability limit
γ
Min dwell-time vs γ
1e−16+1e−3
τmin [s]
Δt = 0.0010s
γ

Table of Contents

Conclusion

- Observer-based event-triggered control of infinite-dimensional systems
  $\Rightarrow$ well-posedness, exclusion of Zeno and exponential stability established
- Systems governed by (dynamics) operators **not required to be skew-adjoint**
  $\Rightarrow$ we rely on **quasi-dissipativity** property
- Role of dynamic triggering in observer-based PDE setting fundamental $\Rightarrow$ *static triggering rules may be insufficient to guarantee absence of Zeno behaviour*

What has been achieved

- **Finite dimension**: uniform dwell-time $\tau_{\min} > 0$ independent of k proved from

$$\left(\gamma \|\hat{x}(t_{k+1})\|_{\mathcal{H}} + m(t_{k+1})\right)^2 \leq 2(t_{k+1} - t_k) \left\|\gamma \|\hat{x}(\cdot)\|_{\mathcal{H}} + m(\cdot)\right\|_{L^\infty(t_k, t_{k+1})} \left\|\dot{\hat{x}}\right\|_{L^\infty(t_k, t_{k+1}; \mathcal{H})}, \tag{37}$$

- **Infinite dimension and spectral decomposition**: uniform dwell-time when **unstable subspace $\mathcal{H}_u$ is finite-dimensional** (parabolic systems)

Open problem: general infinite dimension

When $\mathcal{A}$ generates a C_0-semigroup with **no spectral gap** bound on $\|\dot{\hat{z}}(t_k)\|$ grows like $e^{ct_k} \to +\infty$: **only No-Zeno is currently provable, not a uniform dwell-time**

Perspectives - Relaxing quasi-dissipativity via a P-norm

What has been achieved

Assumption 23 requires a quasi-dissipativity property in $\mathcal{H}$-norm can be weakened
$\Rightarrow$ it exists a Lyapunov quadratic functional $V : \mathcal{H} \to \mathbb{R}_+$ associated to $\mathcal{A} + BK$ such that

$$\exists m_1, m_2 > 0, \quad m_1 \|z\|_{\mathcal{H}}^2 \leq V(z) \leq m_2 \|z\|_{\mathcal{H}}^2, \quad \forall z \in \mathcal{H}, \tag{38}$$

$$\exists P \in \mathcal{L}(\mathcal{H}), \quad P = P^*, \qquad V(z) = \|Pz\|_{\mathcal{H}}^2, \quad \forall z \in \mathcal{H}, \tag{39}$$

$$\exists \beta > 0, \quad \forall z_0 \in \mathcal{D}(\mathcal{A}), \quad \frac{d}{dt} V\left(e^{t(\mathcal{A}+BK)} z_0\right) \leq -2\beta V\left(e^{t(\mathcal{A}+BK)} z_0\right), \quad \forall t \geq 0, \tag{40}$$

Open directions

- **Non-self-adjoint** P: multiplier-based Lyapunov functionals for hyperbolic systems framework allows $P \neq P^*$, but *constructing Gramian in this case requires new admissibility results*
- **Boundary observation** (C_{obs} unbounded): extend Gramian framework to *Salamon-Weiss well-posed systems class*

Perspectives: Triggered observations

Two distinct sampling problems

So far: control $u(t) = K\hat{z}(t_k)$ is sampled, observation $y(t) = Cz(t)$ is **continuous**

Two natural extensions:

1. **Triggered observations, continuous control**: y is only transmitted at instants $\{\tau_j\}$, control $u(t) = K\hat{z}(t)$ is updated continuously using observer
2. **Both triggered**: control sampled at $\{t_k\}$, observation transmitted at $\{\tau_j\}$, two independent or coupled trigger rules

What changes

- Observer error $e(\cdot)$ no longer satisfies a simple ODE: between two observation instants, $\dot{e} = (\mathcal{A} + LC)e$ but L acts on a stale output $y(\tau_j)$, not on $y(t)$

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Assumption (Riesz spectral decomposition)

Operator $\mathcal{A}\colon \mathcal{D}(\mathcal{A}) \subset \mathcal{H} \to \mathcal{H}$ *is a Riesz-spectral operator with discrete spectrum*

$$\sigma(\mathcal{A}) = \{\lambda_n\}_{n\geq 1}, \quad \lim_{n\to\infty} \mathrm{Re}(\lambda_n) = -\infty,$$

Fix a threshold $\lambda_c > 0$ *and define*

$$\sigma_u \coloneqq \{\lambda \in \sigma(\mathcal{A}), \quad \mathrm{Re}(\lambda) \geq -\lambda_c\}, \quad N \coloneqq |\sigma_u| < \infty, \quad \sigma_s \coloneqq \sigma(\mathcal{A}) \setminus \sigma_u, \tag{41}$$

Let $P_u, P_s \in \mathcal{L}(\mathcal{H})$ *be corresponding Riesz spectral projectors satisfying*

$$P_u + P_s = I, \quad P_u P_s = P_s P_u = 0,$$

$$\mathcal{H}_u \coloneqq P_u \mathcal{H} \cong \mathbb{C}^N, \quad (\textit{finite-dimensional}), \quad \mathcal{H}_s \coloneqq P_s \mathcal{H}, \quad (\textit{infinite-dimensional}), \tag{42}$$

$$\mathcal{A}_u \coloneqq A|_{\mathcal{H}_u} \in \mathcal{L}(\mathcal{H}_u),$$

and generator of an (analytic) exponentially stable semigroup on $\mathcal{H}_s$,

$$\mathcal{A}_s \coloneqq A|_{\mathcal{D}(\mathcal{A})\cap\mathcal{H}_s},$$

satisfying

$$\exists M_s \geq 1, \quad \exists \alpha_s > \lambda_c > 0, \quad \|\exp(t\mathcal{A}_s)\|_{\mathcal{L}(\mathcal{H}_s)} \leq M_s e^{-\alpha_s t}, \qquad t \geq 0, \tag{43}$$

Assumption (Mb)

- $m \in C^0(\mathbb{R}_+; \mathbb{R}_+)$,
- $\forall T > 0, \quad \exists \delta_T > 0, \quad \inf_{t\in[0,T]} m(t) \geq \delta_T$,
- $\exists C_m > 0, \quad \exists \mu > 0, \quad m(t) \leq C_m e^{-\mu t}, \quad \forall t \geq 0$,
- $\exists C_e > 0, \quad \forall T \geq 0, \quad \sup_{t\in[T,T+1]} \|LCe(t)\|_{\mathcal{H}} \leq C_e m(T+1)$;

Instance 1: Exponentially decaying $m(\cdot)$.

$$\dot{m}(t) = -\mu m(t) + \zeta \left\| Ce(t) \right\|_{\mathcal{Y}}, \quad \forall t \geq 0 \quad m(0) = m_0 > 0, \quad \mu \in (0, \alpha_0), \tag{44}$$

$\Rightarrow$ yields exponential stability up to a at decay rate (in squared norm) $2\lambda = \min\{2\rho_\gamma, 2\alpha_0, 2\mu\}$ and excludes Zeno behaviour

$\Rightarrow$ uniform dwell-time in suitable settings

Instance 2: Constant $m(\cdot)$.

Choose (constant)

$$m(\cdot) \equiv m_0 > 0, \tag{45}$$

$\Rightarrow$ but practical stability: the observer state $\hat{z}(\cdot)$ converges not to zero but to a neighbourhood of the origin whose size is proportional to m_0
$\Rightarrow$ dwell-time is uniform in suitable settings
$\Rightarrow$ appealing for implementation since $m(\cdot)$ requires no dynamics, but it sacrifices asymptotic convergence to zero

Choose for some $m_\infty > 0$, $m_0 > m_\infty$ and $\mu > 0$

$$m(t) = m_\infty + (m_0 - m_\infty)e^{-\mu t}, \tag{46}$$

$\Rightarrow$ again practical stability to an attractor of size proportional to m_∞
$\Rightarrow$ uniform dwell-time $\tau_{\min} > 0$ (independent of k) in suitable settings
$\Rightarrow$ allows tuning the trade-off between convergence accuracy (taking m_∞ small implies smaller attractor set) and sampling frequency ($\tau_{\min}$ increases with m_∞)

Instance 4: Polynomially decaying $m(\cdot)$.

Choose for some $\alpha > 0$

$$m(t) = m_0(1+t)^{-\alpha}, \tag{47}$$

$\Rightarrow$ stability estimate yields a polynomial decay (rather than exponential) giving

$$\|\hat{z}(t)\|_{\mathcal{H}} \lesssim (1+t)^{-\alpha}, \quad \text{for } t \text{ large} \tag{48}$$

$\Rightarrow$ may still be acceptable in applications where exponential stability is not required
$\Rightarrow$ a uniform dwell-time cannot be guaranteed without a lower bound on $m(t)$ that is uniform in t (not just in T)